Modern human PET-scanners often have high scatter fractions due to the lack of septa. This work tried to treat the scatter correctly by calculating the system matrix elements directly with Monte-Carlo (MC) methods. A parametric compression method was used to scale down memory consumption and the resulting images (reconstructed with either the compressed or with the uncompressed MC-matrix) were compared.

The simulations were performed by a dedicated ring-PET MC-code which is in parts derived from the MC-code XVMI used in radiotherapy [1]. Detection forcing and forced non-absorption, similar to the corresponding variance reduction techniques in SimSET [2], have been used. The simulations were performed in voxelized density phantoms using linear attenuation coefficients calculated from the density information as described in [1].

The system matrix was filled with data obtained by successive MC sub-simulations. Each sub-simulation started a defined number of photon pairs randomly positioned in a voxel. The obtained sinograms for each voxel \( j \) formed columns of the system matrix \( P_j \) where \( j \) represents a line of response (LOR). The rows (constant \( \phi \), variable \( \rho \) in a sinogram of a single voxel \( j \) were modeled by functions \( f_{j}(\rho) \). These functions were composed out of three parts: a central discrete part and two lateral mono-exponentials:

![Figure 1: Compression of a row of an off-central voxel.](image)

The left and right slopes were fitted in the log-plot by a least squares method to straight lines, assuming Poisson statistics (see Fig. 1 and Fig. 2).

![Figure 2: Uncompressed (left) and compressed (right) sinogram of an off-central voxel (windowed gray-values to show scatter).](image)

The simulated scanner had a diameter of 96 cm and consisted of one ring with 4 cm depth and 384 detectors. The larger than usual depth was used to obtain better statistics. The detectors of the scanner were considered to be ideal: 100% efficiency, curved along the nappe of the cylinder, with no depth and without inter-detector or inter-crystal spacing. An energy cut-off for energies below 300 keV was assumed for the detectors.

As a phantom a water filled cylinder (\( D = 30 \text{ cm}, 4 \text{ cm long} \)) with an L-shaped hole was used. The cylinder was filled with uniform activity, whereas two areas were left without activity and other two areas were filled with \( 4 \times \) and \( 2 \times \) the background activity (see Fig. 3 and Fig. 4).

![Figure 3: Density](image)

![Figure 4: Activity (ratio 0.124).](image)

The whole volume to be reconstructed was \( 40 \times 40 \times 4 \text{ cm}^3 \) (\( 64 \times 64 \times 1 \text{ voxel} \)). The images were reconstructed using maximum likelihood expectation maximization (ML-EM) without any regularization. The reconstruction started with uniform activity in all voxels.

Using \( 5 \times 10^7 \) started photon pairs for the simulation of the sinogram of the phantom and \( 5 \times 10^7 \) started photon pairs for each voxel of the system matrix, the following reconstructed images were obtained (20 iterations ML-EM):

![Figure 5: Reconstructed with uncompressed matrix.](image)

![Figure 6: Reconstructed with compressed matrix.](image)

The image reconstructed with the compressed matrix was more grainy (see Fig. 5 and Fig. 6). Probably some rows in the matrix were not fitted well. Looking at a compressed sinogram of the matrix (Fig. 2) revealed that a fit/compression in \( \phi \) direction as well might solve this problem.

![Figure 7: Vertical profile.](image)

A vertical (Fig. 7) and horizontal (Fig. 8) profile through the middle of the reconstructed activity distributions were taken. The less smooth profiles through the activity reconstructed with the compressed matrix (Fig. 6) confirm the grainy impression of this image. For both matrices, the higher activity concentrations (\( 4 \times \) and \( 2 \times \) background activity) matched very well whereas the zero activity regions reached only approximately 25% of the background activity in air (lower L-shape) and even only 50% in water (upper L-shape). Outside the cylinder there was virtually no activity reconstructed.

The proposed fitting-compression scheme did reduce the matrix size about a factor of 26.

The system matrix was calculated using the the PVM (parallel virtual machine)-library on 14 2.8-GHz-processors. The calculation time was about 22 h.

Images reconstructed with MC-based system matrices and ML-EM are rather smooth and activity artefacts outside the phantom are not present at all. Compression of the matrix did not improve the image quality, but resulted in more grainy images. Older simulations with worse statistics, however, showed that compression improved image quality, especially smoothness. As a consequence one can conclude that a better two dimensional compression algorithm could do the same in this case as well. Further research to improve the compression scheme as well as the MC-code is therefore worthwhile.

![Figure 8: Horizontal profile.](image)

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### References
